Further application of a semi-microscopic core-particle coupling method to the properties of ^{155,157}Gd and ^{159}Dy

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Abstract

In a previous paper a semi-microscopic core-particle coupling method that includes the conventional strong coupling core-particle model as a limiting case, was applied to spectra and electromagnetic properties of several well-deformed odd nuclei. This work, coupled a large single-particle space to the ground state bands of the neighboring even cores. In this paper, we generalize the theory to include excited bands of the cores, such as beta and gamma bands, and thereby show that the resulting theory can account for the location and structure of all bands up to about 1.5 MeV.

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1 Introduction

In a previous paper [1, 2] we have applied the Kerman-Klein approach [3, 4, 5] in the adaptation of Dönau and Frauendorf (KKDF) [6, 7, 8] to study rotational bands of odd deformed nuclei, specifically for the nuclei ^{157,159}Gd and ^{159}Dy. This was done by coupling a phenomenological rigid rotor, which was described by the Bohr-Mottelson model, to a single particle. In these applications, only the ground-state band of the cores was included. The systems were described by the conventional monopole pairing plus quadrupole-quadrupole effective Hamiltonian.

Though the model explored by us is technically more difficult to implement than the standard particle-rotor models, the results we found were sufficiently satisfactory that we have been encouraged to develop a more elaborate version of our work in order to account for remaining discrepancies. Most striking of the successes is that without including any *ad-hoc* Coriolis attenuation factors we were able to reproduce the experimentally observed energy levels and electromagnetic transitions of the lowest bands. Nevertheless, there was a shortcoming for all the applications tried in the previous work, in that for every nucleus one or more observed bands at about 1 MeV

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or higher than the ground state band was missing from the theoretical results. It is apparent that these discrepancies, which we attempt to correct in the present paper, arise from the failure to include appropriate excited bands of the even core.

Another source of concern about our previous work, the simplified nature of the Hamiltonian, will not be investigated in the present work. We remark in passing, however, that some preliminary studies including quadrupole pairing and hexadecapole interactions have been carried out for the cases considered here. We found that these interactions have a small effect on our results if we restrict their strength to be reasonably smaller (an order of magnitude smaller) than the leading interactions (quadrupole-quadrupole and monopole-pairing).

We return to the main thesis of this paper. Looking at the experimentally observed spectra of the even cores for the nuclei under consideration we can see that β, γ and other higher bands occur at low energy ($\simeq$ 1MeV). The aim of our treatment will be to include those excited bands (all highly collective as far as intra-band transitions are concerned) that are observed to have non-negligible transition rates to the ground state band.

The general formalism has been described fully in our previous work, where it was specialized afterwards to the case that the core was represented only by the ground state band. Therefore, in the following, we shall consider in detail only those formulas which require generalization compared to the previous application.

In the following, Sec. 2 will be devoted to the phenomenology of the even cores and to the phenomenological model used to fit the experimental results. In Sec. 3 we shall develop the extension of the KKDF model needed for the inclusion of the excited bands and describe the results of the calculations for ^{155,157}Gd, ^{159}Dy. Finally the last section, 4 will contain our concluding remarks.

2 Phenomenology of the even cores

In the previous work we ignored all excited bands and thus inter-band transitions were absent from the model. Experimental results justify this assumption to lowest order, since inter-band BE(2) values, in the nuclei in which we are interested, are two orders of magnitude smaller than the intra-band transitions, i.e. quadrupole transitions matrix elements are an order of magnitude smaller. This assumption worked well for the low-lying levels (less than 1MeV excitation). Nevertheless, the presence of bands not described by the previous work impel us to include the effects of excited bands.

In Figs. 1, 2 and 3 we display energy spectra together with observed BE(2) values for intra and inter-band transitions of cores used in the present study. As can be seen from the figures all even nuclei have excited bands at about 1 MeV and the inter-band transitions are small but not zero. For example, in the case of ^{156}Gd we include the β, γ and a third excited band at about 1 MeV above than the ground state band. As previously stated the values of the inter-band BE(2) transitions are of the order of 100

smaller than the values of the corresponding intra-band transitions. This is a pattern we see in all the neighboring nuclei. The facts that these transitions are not zero and the fact that these bands occur at relatively low energies justify their inclusion in the calculations.

To perform the calculations for the odd nuclei the excitation energies and the quadrupole operator matrix elements between any two states of the even neighbors that belong to the ground-state band or to one of the excited bands have to be either known from experiment or calculated from a phenomenological model. Since there are not enough experimental values to cover all our needs, we have to use a phenomenological description to calculate the transition matrix elements and the excitation energies not available experimentally, i.e., we use the phenomenology only to augment experimetal information.

For the excitation energies we found it sufficient to use the simple formula,

$$\omega_{IK} = E_K + \frac{\hbar^2}{2\mathcal{I}_K}(I(I+1)),\qquad(1)$$

where E_K is the band-head energy and $\mathcal{I}_K$ is the moment of inertia of the given band. In Fig. 4 we show the results of fitting to this formula. We see that this simple formula is sufficient for the reproduction of the experimental results, provided we adjust $\mathcal{I}_K$, the individual values differing from each other by up to 20%.

For the transitional matrix elements we again use the phenomenological description given by the geometrical model of Bohr and Mottelson, applicable either to intra or inter-band transitions,

$$\langle IK\|Q\|I'K'\rangle = \sigma_{K'}q_0^{\text{band1}\to\text{band2}}\sqrt{(2I+1)(2I'+1)}\begin{pmatrix} I & 2 & I' \\ K & -(K-K') & K' \end{pmatrix},\qquad(2)$$

where $\sigma_{K'}$ takes the value 1 if $K' = 0$ and $\sqrt{2}$ if $K' \neq 0$. In Fig. 5 we show results of fitting using the above equation to the experimental BE(2) values for ^{156}Gd. Similar fits were performed for all neighboring nuclei. It is clear that the agreement is far from ideal. We emphasize however that experimental values, whether for energy or matrix elements were used whenever available, and the phenomenological values were utilized only in the absence of the former (with some smoothing enforced at the boundary between known and unknown values).

3   Calculations

Here we present the method and results for energy states in ^{155}Gd, ^{157}Gd and ^{159}Dy, obtained by coupling a large single-particle space (single-particle states from 5 major
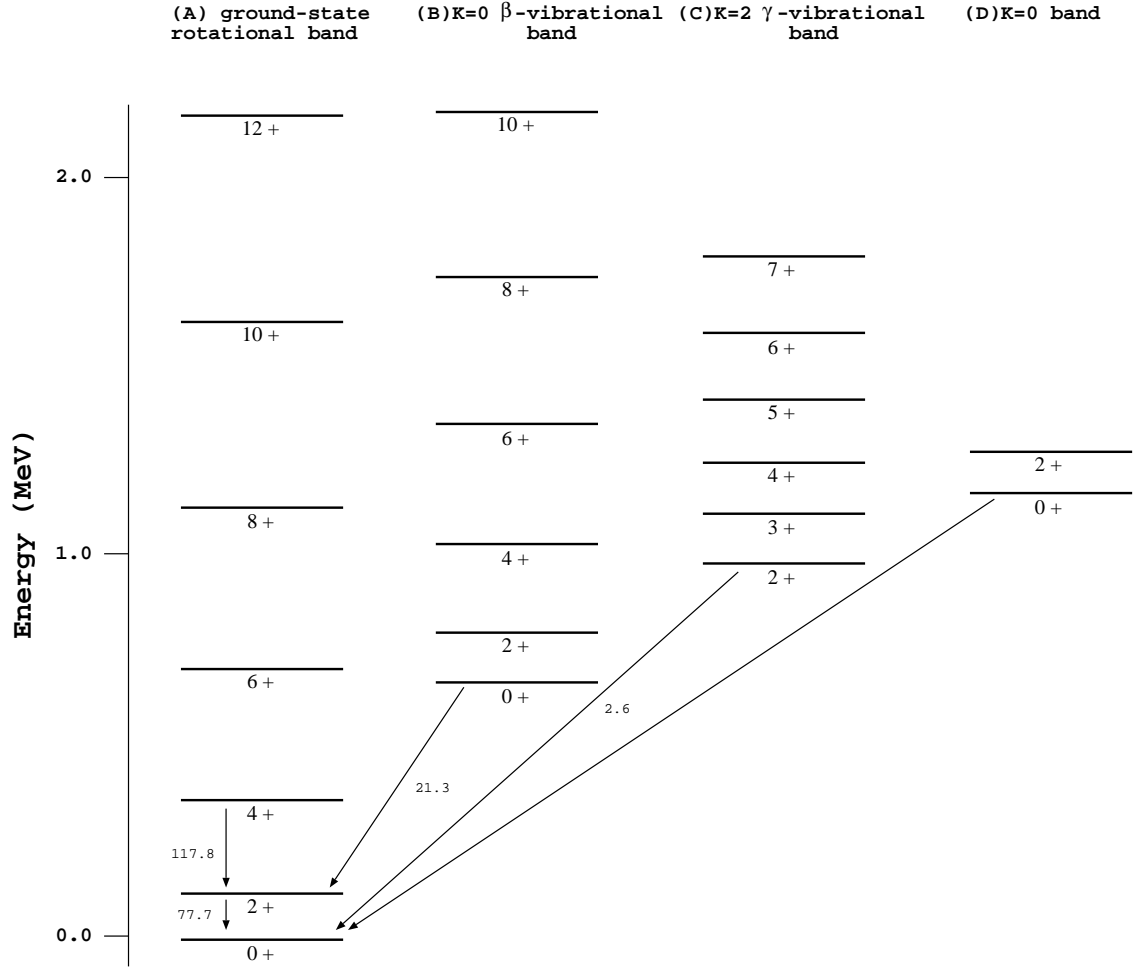
Gd154

Figure 1: Energy levels and some BE(2) values for ^{154}Gd. The BE(2) values are given in $[10^{-2}(eb)^2]$. Experimental values for the BE(2)'s are taken from Ref. [9]
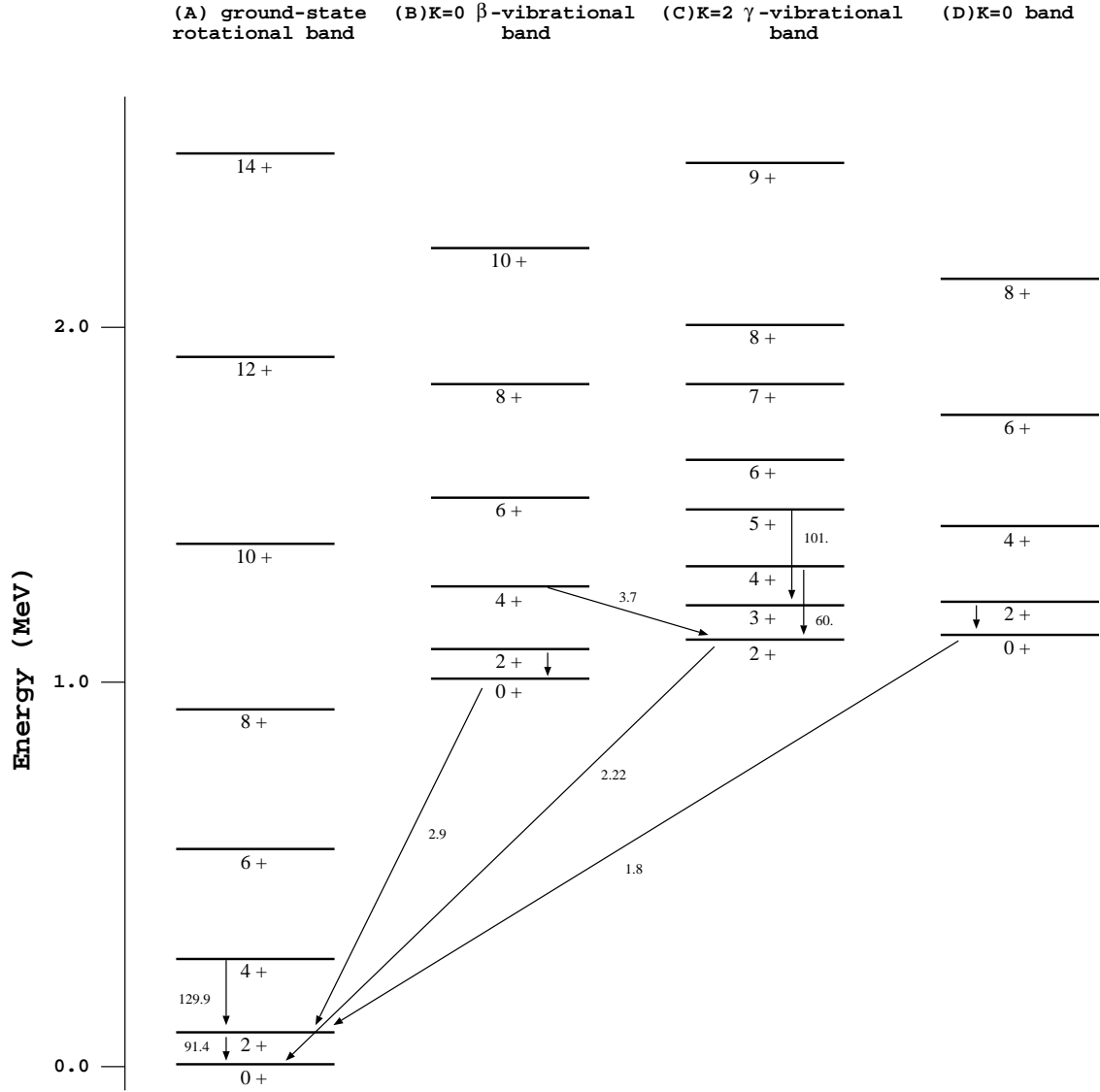
Gd156

Figure 2: Energy levels and some BE(2) values in $[10^{-2}(eb)^2]$ for ^{156}Gd. Experimental values for the BE(2)'s are taken from Ref. [9]
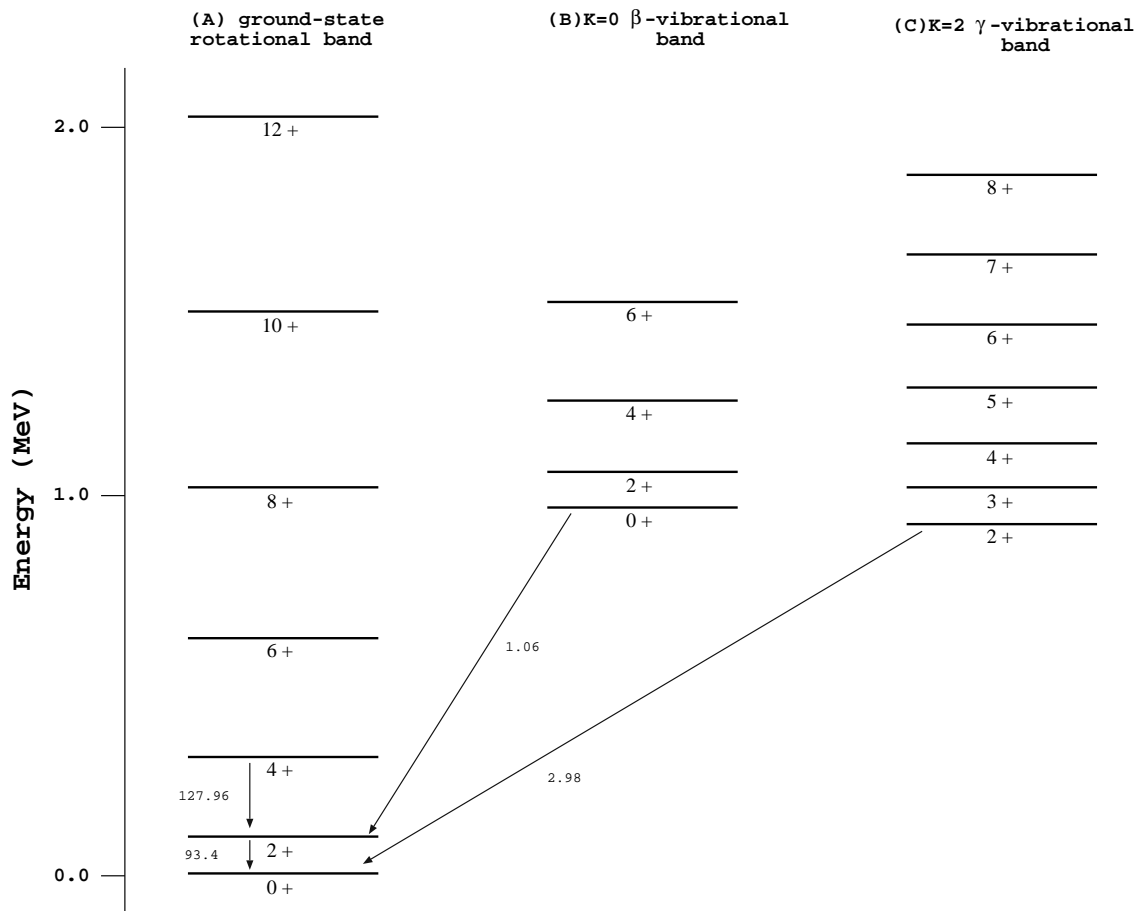
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Figure 3: Energy levels and some BE(2) values in $[10^{-2}(eb)^2]$ for ^{158}Dy. Experimental values of BE(2)'s are taken from Ref. [10]
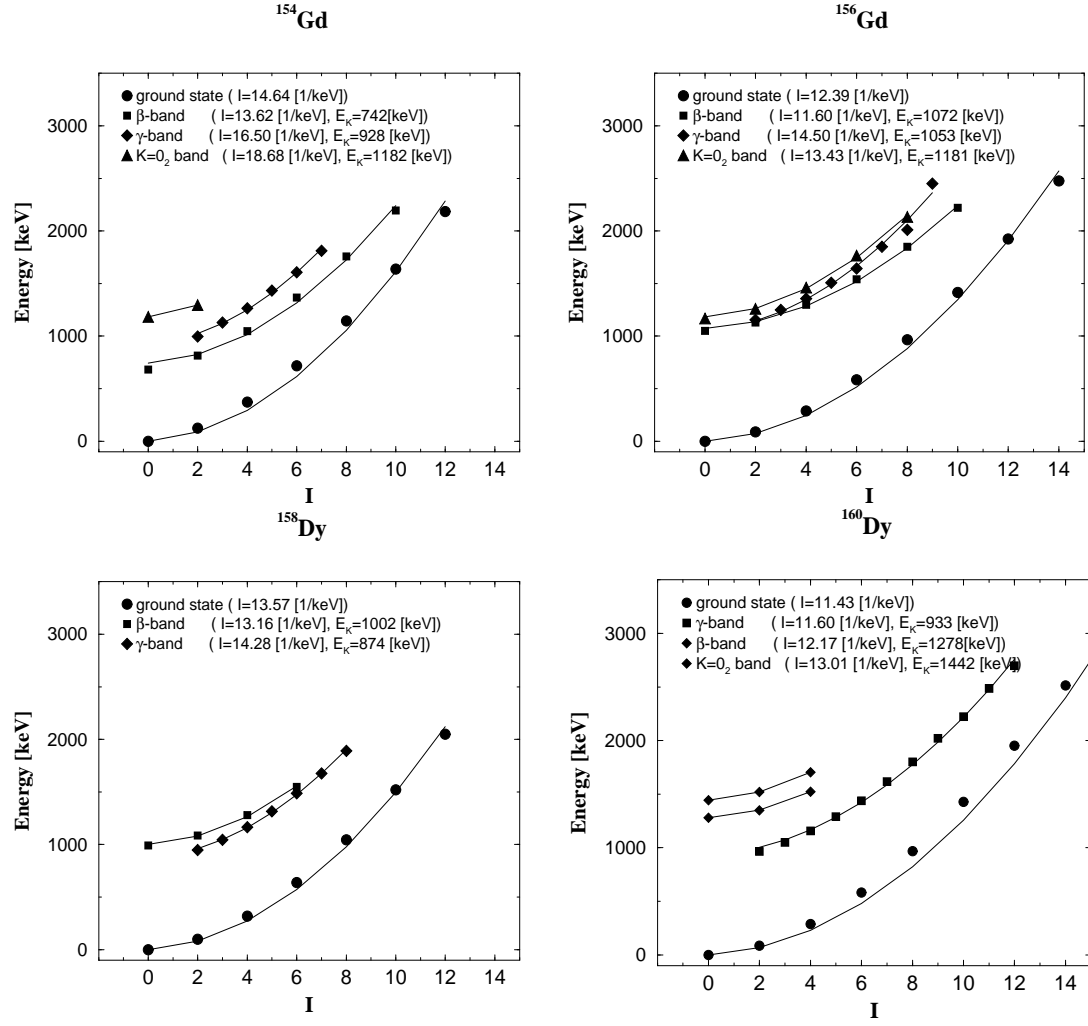
Figure 4: Experimental and fitted energy levels of ^{154}Gd, ^{156}Gd, ^{158}Dy and ^{160}Dy. Experimental values are shown in filled symbols and the solid lines represent the theoretical values. The band-head energies and the values of the moments of inertia of the corresponding bands are also given for each band.
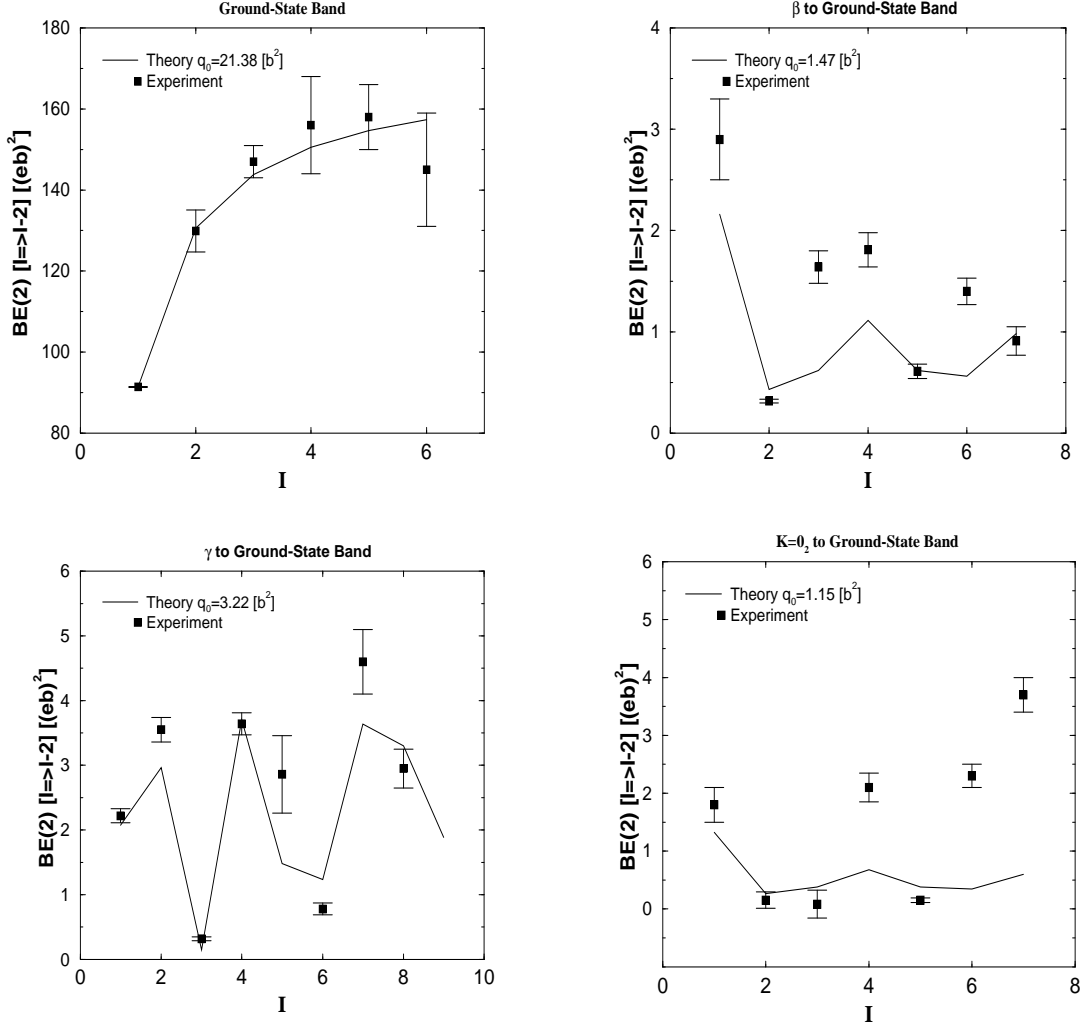
^{156}Gd

Figure 5: BE(2) values for ^{156}Gd. The experimental values are shown as filled squares and the theoretical predictions are the solid lines. For every intra or inter-band transitions we give the fitted intrinsic quadrupole moment q_0.

shells were included) to the ground and excited bands of the appropriate neighboring even nuclei, using the average description of the latter implied in the number non-conserving approximation. As stated above, the underlying theory, equations of motion, etc, are the same as that described in detail in our previous work, and therefore will not be repeated here. As also noted, all required excitation energies or transitions (quadrupole transitions) of the even cores were either calculated or taken from experiment. The matrix elements of the even cores were expanded to include transitions outside the ground-state band according to the formula for the core-particle quadrupole interaction, Γ,

$$
\begin{aligned}
\Gamma_{aIK,cI'K'} &= -\sigma_{K'}\ \kappa\ q_0^{\text{band1}\rightarrow\text{band2}}(-)^{j_c+I+J} \\
&\times \begin{Bmatrix} j_a & j_c & 2 \\ I' & I & J \end{Bmatrix} \sqrt{(2I+1)(2I'+1)} \begin{pmatrix} I & 2 & I' \\ K & -(K+K') & K' \end{pmatrix} q_{ac},
\end{aligned}
\tag{3}
$$

where κ is the strenght of the quadrupole-quadrupole interaction and the core excitations take the form of Eq. (1) to accommodate the excited bands. The values of $q_0^{\text{band1}\rightarrow\text{band2}}$, E_K and $\mathcal{I}$ were calculated as described in the previous section.

The Hamiltonian matrix for the odd nuclei, to which the theory gives rise, and which now includes the excited bands, is decomposed into two parts (in the same fashion as in the previous work) one which is antisymmetric with respect to particle hole conjugation (for which physical and unphysical solutions clearly separate into solutions with positive and negative energies, respectively) and the other of which is symmetric. First we diagonalize the antisymmetric part and identify the physical solutions (those with positive energy). Then the symmetric part is turned on "slowly" and at every step the physical solutions were identified by checking the eigenvalues of a projection operator which is built from already identified physical wavefunctions of the previous step. As soon as the steps of the iteration are small enough to guarantee that the wavefunctions do not change rapidly between two steps, this procedure works very well, as it did in our previous work. This is because the physical solutions have expectation values close to unity and the unphysical ones values close to zero.

Nevertheless, it is known that for the case that two states with the same angular momentum come close to each other, they repel and never really cross (thus the name "avoided crossing") (see Fig. 6). Furthermore it is known from sufficiently general model studies that the wavefunction for each of the bands changes rapidly in the neighborhood of the crossing and the two end up inter-changing their character after the crossing. For example in Fig. 6 before the crossing the states are almost equal to the uncoupled states $|1,0\rangle$ and $|0,1\rangle$. After the crossing the state that corresponded to $|1,0\rangle$, is now $|0,1\rangle$ and the state that corresponded to $|0,1\rangle$ is $|1,0\rangle$.

Because the excited bands have band-head energies of about 1 MeV the possibility of two states coming close to each other is much higher in the present case than in our previous work. In order to make sure that the procedure described above (the projection operator method) works, we have to make the steps extremely small; as a

result the numerical procedure becomes extremely slow. To avoid this slowdown, we have developed a special stratagem. At two successive steps, the program checks if there was any kind of crossing by inspecting the differences of eigenvalues of any two eigenstates, identified as physical or unphysical according to the standard method. If the sign of the difference changes between two steps, then a crossing has occurred. (In the case that a crossing is detected, the projection operator would identify a physical state as unphysical since, as explained above, their wavefunctions have been interchanged.) Since we know that every time there is a crossing it is an avoided crossing, when a crossing is detected we classify a state as physical which otherwise would have been identified as unphysical from the projection operator method. Finally the new projection operator will be built from the new physical wavefunctions. Therefore in the subsequent steps, we can continue with the projection operator method.

Another technical problem is the classification of states into different bands. This was done the same way as in our previous work, but we include a brief and hopefully clear explanation of the procedure.

Recalling that our formalism remains rotationally invariant even after omitting the excitation spectrum of the even neighbors, we start in this limit with a $J = \frac{1}{2}$ submatrix calculation. The distinct solutions are identified as $K = \frac{1}{2}$ bands. These levels reappear for all higher J calculations at the *same* energy. Thus for $J = \frac{3}{2}$, additional solutions are identified as $K = \frac{3}{2}$ bands, etc.
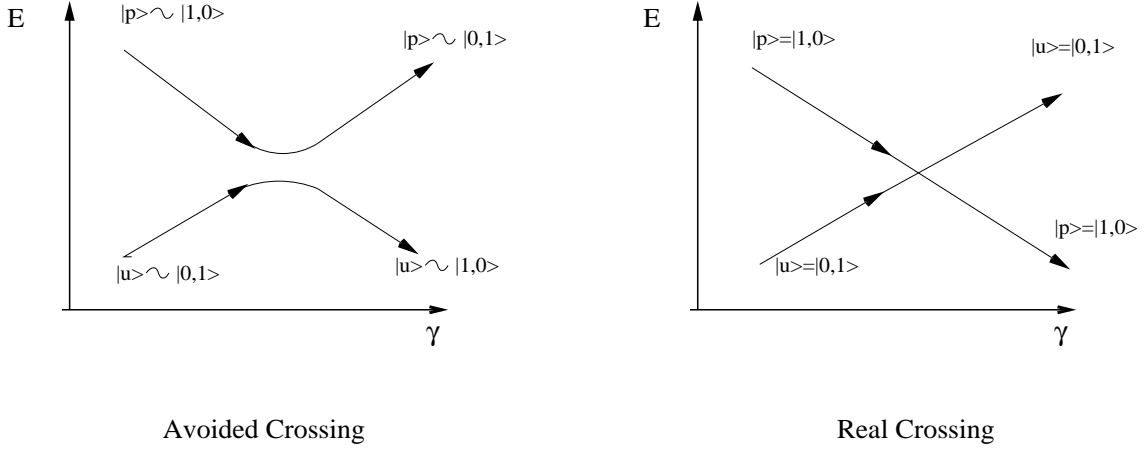

Figure 6: Schematic representation of a real crossing (on the right)   and an avoided crossing (left).

In Figs. 7,8 and 9 we show results of the calculations for three odd nuclei ^{155}Gd, ^{157}Gd and ^{159}Dy. In the case of ^{155}Gd a $K = 1/2$ band at about 0.6 MeV was not reproduced in the calculations without excited bands, refered to as phonons here.

When phonons were included in the calculations the missing band was reproduced at the right band-head energy and almost right band structure. The structure of all other bands and their relative band-head energies have not changed more than one percent. This is because the off-diagonal elements of the Hamiltonian are very small (value of the inter-band transitions) compared to the diagonal elements. The strength of the quadrupole force was treated as a free parameter and was fitted to experimental band-head energies. The best fit was achieved for $\kappa = 0.380$ MeV/fm^2 which is almost equal to the strength found in the case of no-phonons (0.377MeV/fm^2). In the case of ^{157}Gd a $K = 3/2$ band at about 0.7 MeV was not reproduced when phonons were not included. With the addition of phonons the band was found at the right band-head energy, and it has the right band structure. As in the case of ^{155}Gd the strength of the quadrupole force is almost the same as for the non-phonon calculations $\kappa = 0.401$ MeV/fm^2 (compared to $\kappa = 0.397$ MeV/fm^2). In the last application we considered ^{159}Dy. When only the ground state band is allowed, two experimentally observed bands were not reproduced: $K = 3/2$ at about .7 MeV and $K = 5/2$ at about 1 MeV. When a β and γ excited bands were included in the calculations both bands were calculated at almost the right band-head energies and have the right band structure.

4 Conclusion

Even though the inclusion of the vibration excitation of the even cores makes the calculations more complicated and numerical solution longer and more tedious the results supply the previously missing bands. It thus seems necessary to include the low-lying excited bands of the neighboring cores in order achieve a good fit to experimental values. The inclusion of the vibrational bands of the even cores does not affect the one quasiparticle plus ground-state bands of the odd nucleus, but some of the low-lying bands are apparently one quasi-particle plus excited core bands.

5 Acknowledgement

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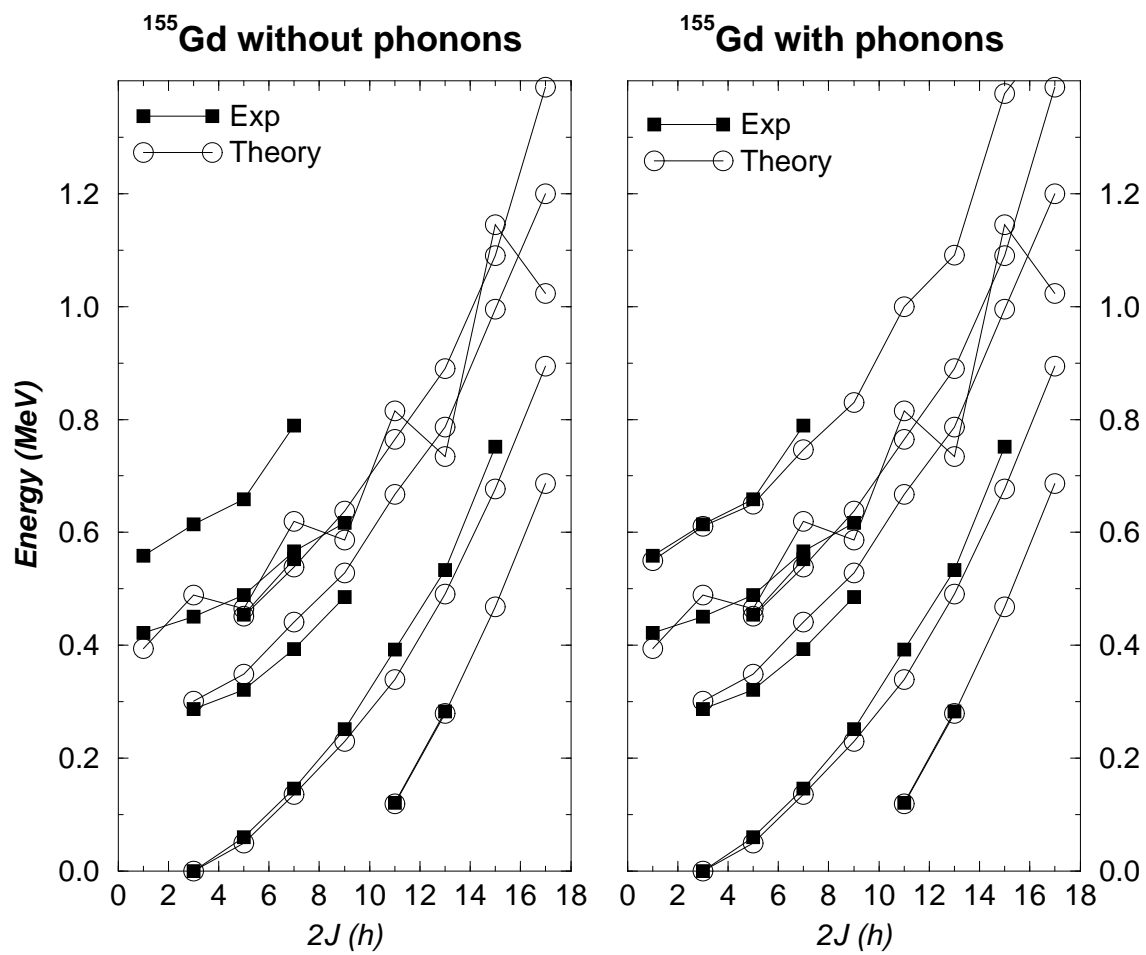[2] P. Protopapas, Ph. D. thesis, U. of Pennsylvania, Sept., 1995, unpublished.

Figure 7:   Energy levels for ^{155}Gd
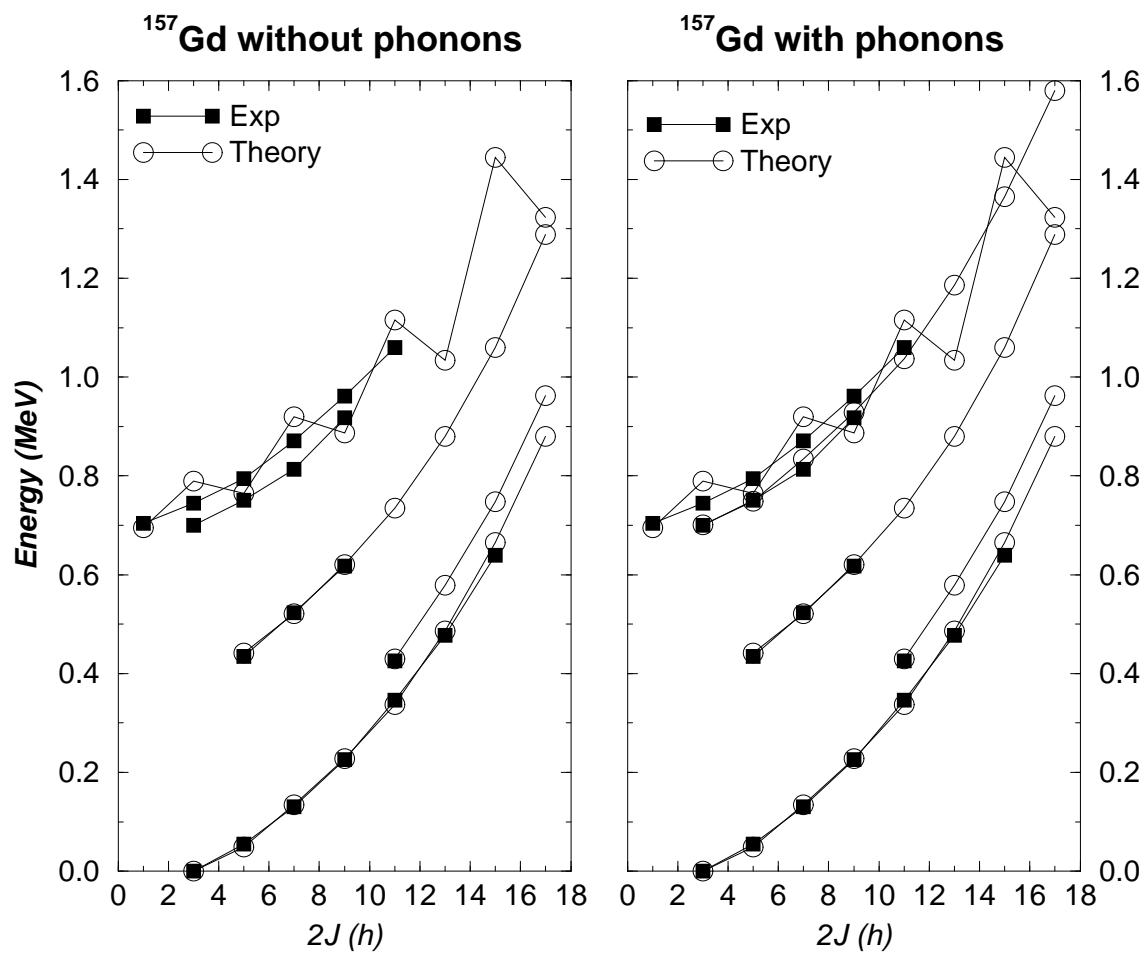
Figure 8:   Energy levels for [157]Gd

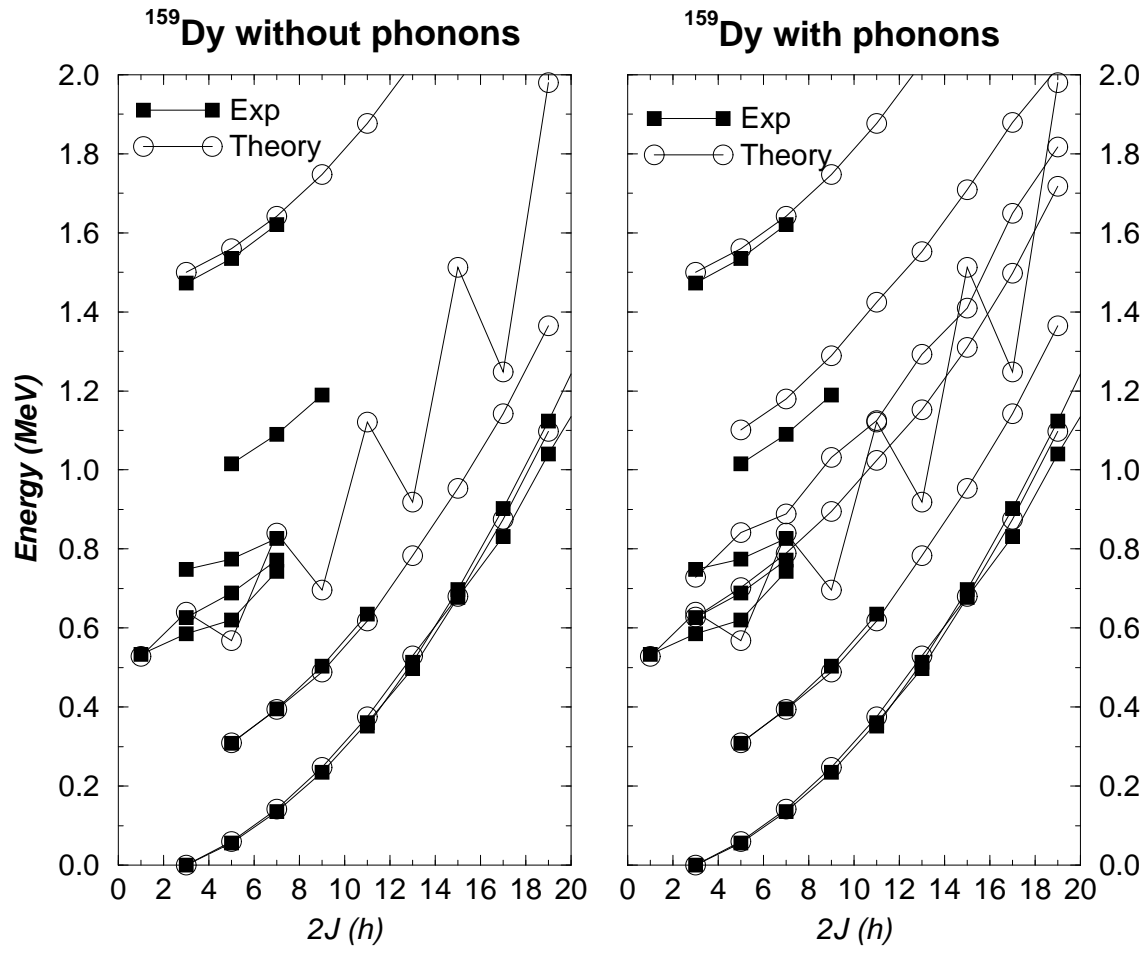Figure 9:   Energy levels for [159]Dy

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